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Towards a Better Understanding of Rotating Turbulent Convection in Geo- and Astrophysical Systems

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Turbulent rotating convection occurs in many geo- and astrophysical bodies, but it is still far from being well understood. Here, we review some recent findings in this field which were obtained using the JUQUEEN system. We present results revealing that tiny viscous boundary layers, so-called Ekman layers, are much more important in rapidly rotating convection than previously thought. We also discuss evidence for upscale kinetic energy transport generating large-scale, coherent structures in rotating convection. Finally, we briefly discuss the geo- and astrophysically relevant case of convective systems in which the fluid parcels in the deeper parts of the convective region get compressed significantly by the weight of the overlying fluid. We show that in the presence of rotation, such compressibility effects can drive alternating jets similar to those observed on Jupiter and other giant planets.

1 Introduction

Buoyancy driven fluid flows, so-called convective flows, are ubiquitous in nature. They stir the outer layer of the sun and play an integral part in transporting the energy generated in the solar interior to the surface, from where it is radiated into space. Convection also occurs in giant planets, and has been proposed to drive the strong zonal winds which organise Jupiter's colourful clouds into the banded structures that dominate the planets' visual appearance. Deep below our feet, the Earth's magnetic field is generated by convective flows in its liquid outer iron core. Convection is also important in daily life and in technical applications, for example when it cools down a cup of tea or when it generates comfortable living room temperatures by redistributing the thermal energy of a radiator.

Geo- and astrophysical convective systems are however special in a number of respects. In particular, *(i)* they typically have a huge spatial extent, *(ii)* they usually spin rapidly, and *(iii)* the fluid in deeper parts of their convective regions can be compressed significantly by the overlying weight. All these features affect the dynamics considerably. The huge system size makes viscous diffusion irrelevant on most flow scales of interest, causing intense turbulence. At the same time, the motions usually feel the effects of rotation, and in many important applications, Coriolis forces dominate the leading order force balance. Finally, fluid particles can get compressed or expanded as they travel between different depth levels, which changes their moment of inertia. As we will discuss below, this can lead to order one changes in the overall dynamics, especially in rapidly rotating systems such as Jupiter, where it is possibly this process driving the zonal winds.

The goal of the research presented here is to gain a better understanding of the basic dynamics of turbulent, rotating convection and of its role in natural systems. In this contribution, we review some recent findings in this field which have been obtained using the JUQUEEN system in Jülich. The presentation style is chosen to appeal to a wider audience, with more details being available in our recent publications[1–7].

2 Modelling Approach and Applied Methods

Numerical simulations in geo- and astrophysics are often tailored to mimic a specific natural system as closely as possible. All physical processes expected to be relevant are included, and complicated geometries and boundary conditions are adopted. While results from such models can be compared directly with observations, the inherent complexity sometimes obscures the view on the fundamental physics. In this work we therefore focus deliberately on conceptually simple models in plane, Cartesian geometries. A rotating plane layer, heated from below and cooled from above with gravity pointing downwards is considered. This choice of setup allows us to directly relate the results to laboratory experiments which are usually carried out in a similar configuration. Furthermore, the most advanced theoretical models are generally developed for this particular setup. Testing the available theory with numerical simulations allows us to asses our current level of understanding and, as we will show, can also guide future theoretical advances.

Three different levels of approximation are used in our simulations, which in increasing order of complexity are (i) the Boussinesq approximation, which accounts for density variations only in the buoyancy force while otherwise treating the flow field as incompressible, (ii) the anelastic approximation, which includes the effects of an adiabatic increase of density with depth, and (iii) the so-called fully compressible model, which contains no approximations beyond those involved in basic continuum-mechanics. The Boussinesq approximation is usually used in the context of laboratory experiments, and also in the bulk of the existing theoretical work. Unfortunately, it breaks down in large-scale natural convection if the material gets compressed significantly by the weight of the overlying fluid. The anelastic approximation takes this effect into account, while maintaining most computational benefits of the Boussineq approach. Its general validity, especially in the context of rapidly rotating flows, has however been called into question recently[8]. The fully compressible model provides the most comprehensive description, and remains valid even under super-sonic conditions and for intense thermodynamic fluctuations. An in-depth discussion, including the relevant equations, is beyond the scope of this overview paper and can be found elsewhere[3,6].

Different codes[9,6] are used for the simulations. In the Boussinesq and anelastic case, high order spatial discretisation schemes are employed, based either on spectral Fourier and Chebychev expansions, or on a hybrid spectral / finite-difference formulation. In the fully compressible case, a second order finite difference scheme is used. Time integration is performed by linear multistep methods (AB/BDF2 and AB/BDF3). All codes have been shown to scale well on JUQUEEN up to at least 10^5 cores.

3 Asymptotic Behaviour of Rapidly Rotating Convection

Direct numerical simulations (DNS) of geo- and astrophysical convection are usually unable to cover the full range of temporal and spatial scales occurring in nature. This difficulty is well known to occur in the presence of strong turbulence, where eddy sizes often range from the system scales all the way down to the so-called Kolmogorov micro-scales at which molecular viscosity becomes important. Similar problems are, however, also encountered in rotating fluids, where it is the Coriolis effect that creates dynamical processes on a broad range of temporal and spatial scales. An important control parameter in this case is the so-called Ekman number $E = \nu/(2\Omega H^2)$, which is defined as the ratio of the rotation time scale $(2\Omega)^{-1}$ to the characteristic time scale of viscous diffusion H^2/ν across the convective layer. Here, ν denotes the kinematic viscosity, Ω is the rotation frequency and H the layer depth. In planetary cores, E is on the order of 10^{-15}, while even the most advanced simulations only reach down to $E = O(10^{-7})$. Viscosity is therefore massively overrepresented in these simulations. The problem is due to an extreme range of spatial and temporal scales that needs to be resolved at small E. While the system scale is H, convective instabilities occur on $O(E^{1/3}H)$ spatial scales and viscous boundary layers have an $O(E^{1/2}H)$ thickness. The time scale of vertical viscous diffusion is H^2/ν, which is $O(E^{-2/3})$ times longer than the horizontal diffusion time scale across convective instabilities, and $O(E^{-1})$ times longer than the period of certain waves appearing in rotating flows. For E as low at 10^{-15}, these scale disparities are too huge to be resolved. Unfortunately, laboratory experiments also have been unable to probe the low Ekman numbers regime beyond the values attainable in simulations, albeit for different reasons[4,10].

As both simulations and laboratory experiments cannot reach realistic parameter values, the question arises whether they nevertheless represent the physics of natural systems adequately. Any geophysical interpretation of their results clearly requires an extrapolation over many orders of magnitude in Ekman number. This can only be done if (*i*) no further bifurcations or regime transitions are encountered within the covered parameter range and (*ii*) if the relevant scaling relations are known exactly. In essence, the physics in the asymptotic limit $E \to 0$ needs to be understood rigorously, and both numerical simulations and laboratory experiments have to advance into regions of parameter space where clear asymptotic behaviour becomes apparent.

Over the past decade, there has been considerable progress in mathematical studies of the low Ekman number limit employing asymptotic expansion techniques. Detailed quantitative predictions for rapidly rotating plane layer convection are now available[11,12], and these are expected to be valid even in the fully turbulent regime as long as Coriolis forces remain dominant in the force balance. JUQUEEN has allowed us to test these predictions[2], and thus our current understanding of rapidly rotating convection. As the available theoretical studies cover the Boussinesq case with anti-parallel rotation and gravity, we also employed this configuration in our DNS. Fig. 1 shows the computed convective heat transport as a function of the forcing strength for a range of Ekman numbers. As described in the figure caption, clear convergence against the theoretical predictions is found if the boundary conditions are chosen to explicitly suppress lateral shear stresses on the bounding surfaces. However, for rigid, no-slip boundary conditions, as they arise in laboratory experiments or planetary cores, the available theory appears to break down.

The crucial difference between the two cases is that viscous boundary layers, so-called Ekman layers, form in the no-slip case. These are extremely thin structures - for the simulations at $E = 10^{-7}$ shown in Fig. 1, the Ekman layers cover only the outermost $\sim 0.1\%$ of the fluid layer (but still need to be properly resolved, of course). It is surprising that these tiny structures can have such a strong impact on the overall dynamics - in some cases, they increase the heat transport across the convective region by more than 800%! Even more surprising is that Fig. 1 suggests that the Ekman layers *gain* dynamical importance as E is reduced, despite the fact that both their thickness and the secondary flows they induce are known to *decrease* with decreasing E. Our simulation results are also puzzling in the light of mathematical studies conducted back in the 1960s, which irrevocably showed that at

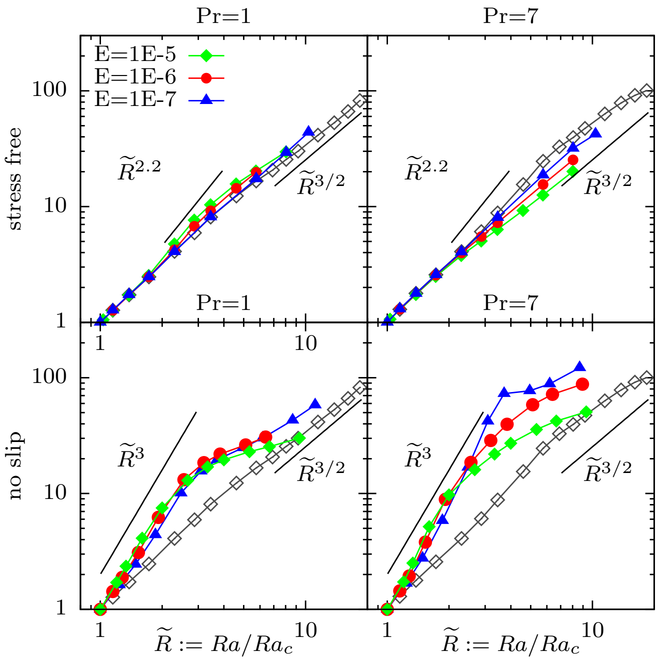

Figure 1. Heat transport, measured in terms of Nusselt number, as a function of super-criticality $\widetilde{R}$. The non-dimensional Nusselt number Nu is defined as the total heat transport normalised by the purely conductive heat transport that would occur in the absence of convection. It is thus a non-dimensional measure of the heat transfer efficiency across the layer. Ra denotes the so-called Rayleigh number, a non-dimensional measure of the driving strength, and Ra_c the value at which convection first sets in. Results are shown for stress-free and no-slip mechanical boundary conditions, and for two different fluids (with Prandtl number $Pr = 1$ and $Pr = 7$), corresponding roughly to air and water. Coloured, full symbols show results obtained in direct numerical simulations carried out on JUQUEEN. Open symbols show theoretical predictions based on asymptotic theory[11,12] expected to be valid at small E. With decreasing E, even in the highly turbulent regime with $Nu \gg 1$ the results converge to the theoretical prediction for stress-free boundary conditions, but not for no-slip boundary conditions. Interestingly, the maximum deviations from the asymptotical predictions increase for no-slip boundaries as the small Ekman number regime is approached.

the onset of convective motions, Ekman layer effects become asymptotically small in the rapidly rotating regime[13]. This suggests that somewhere above onset, a previously overlooked non-linear effect kicks in that brings the dynamical consequences of the tiny Ekman layers back into the leading order dynamics, with enormous effects on the heat flow.

Using an in-depth analysis of the numerical simulations, a refined version of the asymptotic theory which explicitly includes the Ekman layers has been developed[7], and detailed computations based on this theory are currently in progress. Computations on JUQUEEN have thus revealed that a key physical process had been missed in previous theories. They also have provided essential information concerning the nature of the missing pieces. At least in the simple geometries considered here, a much more comprehensive understanding of rapidly rotating convection now appears within close reach.

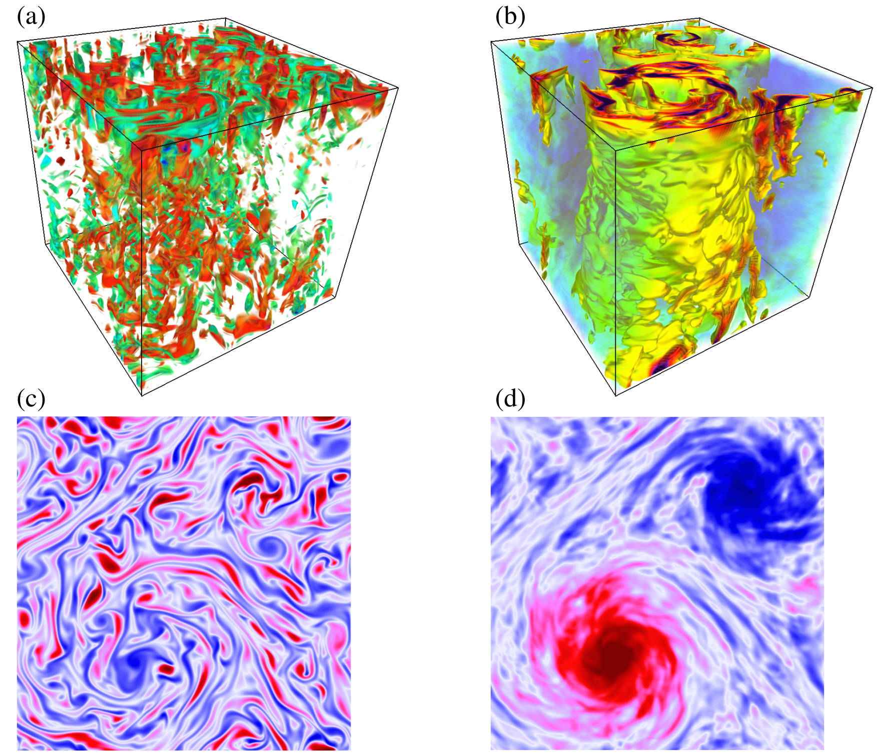

Figure 2. Formation of large-scale, coherent vortices in rotating Rayleigh-Bénard convection ($E = 10^{-7}$, $Pr = 1$ and $\widetilde{R} \approx 10$) with stress-free boundaries. Shown is (a) vertical vorticity, (b) kinetic energy of horizontal flow components, (c) temperature anomaly close to the upper boundary and (d) vertically integrated vertical vorticity. As apparent from panels (a) and (c), the convective flow itself exhibits a small-scale, turbulent, three-dimensional structure, but drives system-scale, coherent vortices as shown in panels (b) and (d). Note that the horizontal scales are stretched by a factor of ~ 4.5 in (a) and (b) to aid the visualisation.

4 Upscale Kinetic Energy Transport in Rapidly Rotating Convection

As stated earlier, convective instabilities in rapidly rotating systems occur on small $O(E^{1/3}H)$ length scales, with H denoting the depth of the convective region. For the Earth's core with $E = O(10^{-15})$, this corresponds to a few tens of meters, which is tiny compared to the outer core radius of roughly 3400km. An important question is whether convective turbulence driven at such small scales is able to generate much larger coherent structures on the global scale H. Such structures could then possibly explain large-scale anomalies in the Earth's magnetic field[5]. Similarly, if the zonal winds on Jupiter and other giant planets are indeed convectively driven, a mechanism needs to exist that transports kinetic energy from the local convective injection scales to the global wind scales[14].

Our simulations on JUQUEEN indeed reveal that system scale coherent structures can form in rapidly rotating Boussinesq convection provided the turbulence level is high enough[2]. An example is shown in Fig. 2. Although the convective flow exhibits many small scale features on the $O(E^{1/3}H)$ convective instability scale, it drives an intense pair of vortices on the largest scale available to the system. The formation of such vortices in rotating convection has recently also been observed in simulations at larger E , where the large-scale energy accumulates predominantly in cyclonic structures[15,16]. Such symmetry-breaking is theoretically predicted to be absent in the asymptotic case of rapid rotation[17]. Indeed, our DNS reveals the generation of both cyclonic and anti-cyclonic vortices, which suggests that symmetry tends to be restored as E is reduced. A further interesting observation is that upscale kinetic energy transport appears to be largely suppressed by no-slip boundaries. An investigation of this effect is currently underway.

5 Modelling Compressibility Effects

So far, we have completely neglected the fluid's compressibility. This is typically justified in laboratory experiments, but not in large-scale geo- and astrophysical systems, where the weight of the overlying fluid often causes a substantial density increase with depth. In Jupiter for example, density varies by about four orders of magnitude from the one bar level down to the region where the atmospheric hydrogen becomes metallic, which is often viewed as a natural lower boundary of an outer convection zone.

There are several ways to model such configurations. One approach is to solve the basic fluid-dynamical equations following from first principles of continuum physics. These equations however also describe sound waves, and the requirement to resolve these can significantly hamper simulations of natural flows which often evolves on much longer time scales. To circumvent this problem, the governing equations can be simplified using the so-called anelastic approximation, which filters out the sound waves. Unfortunately, there are some concerns about the general validity of this approximation, and problems have recently been predicted to occur for low viscosity fluids in rapidly rotating systems[8]. It is therefore necessary *(i)* to check the validity bounds of the approximation, *(ii)* to quantify the arising errors and *(iii)* to quantify the relative computational efficiency for both approaches.

JUQUEEN has recently allowed us to compare both modelling strategies systematically, from the laminar to the turbulent regime, for the first time[3]. As a first step, an ideal gas has been taken as the working fluid, and only the non-rotating case has been considered so far. Our simulations show that results obtained using the full equations converge to the anelstic case exactly as predicted by theory, even in the turbulent regime. In addition, they provide quantitative error estimates as well as rough guidelines concerning the computational efficiency of each approach. This enables modellers to chose the most accurate and efficient approach for their particular problem at hand. We are currently extending this study to the rapidly rotating case.

6 Are Jupiter's Zonal Winds Driven by Compressibility?

Strong zonal winds organise the colourful clouds on Jupiter's surface into characteristic banded structures that can be observed even with basic amateur-level telescopes. Higher resolution pictures reveal that these winds are embedded in a sea of small scale turbulence. Similar pronounced zonal wind structures have been observed on all gas and ice giants in our solar system, and thus appear to be a very robust feature. Despite the tremendous observational progress that has been made over the last decades, the dynamical origin of the

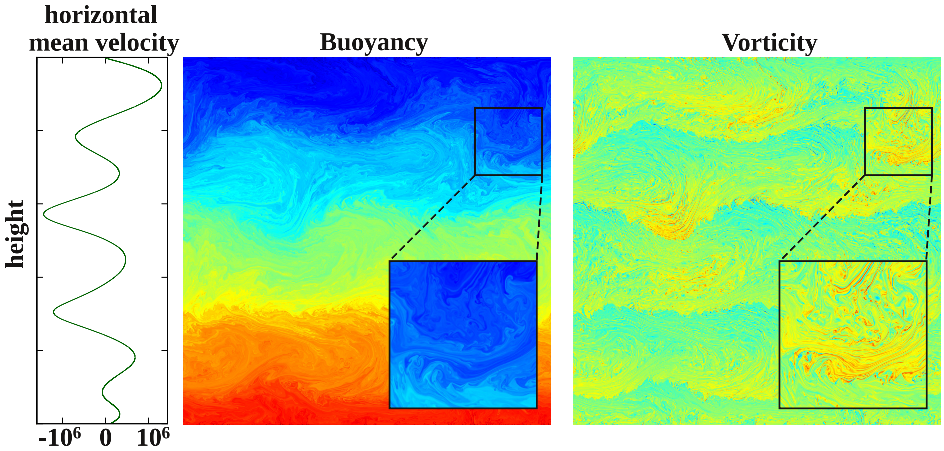

Figure 3. Generation of horizontal jets in a two-dimensional simulation of compressible, rotating convection. A compressible fluid is heated from below and cooled from above, with the rotation axis being perpendicular to the plane of the paper. The relative orientation of gravity and rotation mimics the conditions on the equatorial plane of giant planets. The colour maps range from blue (negatively buoyant, negative vorticity) to red (positive buoyancy and vorticity). Kinetic energy is transported from small-scale convective instabilities to larger spatial scales until the structures become large enough for compressional Rossby waves to be excited, visible here as large-scale, undulating structures. The energy then gets channelled predominantly into strong alternating jets.

winds remains poorly understood. Some models emphasise dynamical effects occurring in the outermost atmospheric layers, invoke approximations valid for shallow atmospheres only and ignore the vast interior of the planet[18]. Other models seek the dynamical roots of the differential rotation in convective motions extending deep downwards into the gaseous envelope[19,14,20]. Today, it is not clear how deeply the jets extend into the planetary interior. The Juno mission that will reach Jupiter in mid-2016 has the potential to better constrain the radial extent of the jets[21].

Here, we focus on the deep dynamics, and investigate the possibility that the jets are driven by compressibility effects. The idea[22] is that in deep planetary convection, an inverse cascade successively transports kinetic energy generated by small-scale convective instabilities to larger scales, in a similar fashion as described in Sec. 4. At some point, the convective eddies become large enough to feel the density increase with depth. When this happens, certain waves, so-called *compressional Rossby waves*, are excited[23]. Their basic dynamics is in many respects similar to the classical Rossby waves that create meanders in Earth's jet stream and influence our mid-latitude weather, but in this case it is the compression of the fluid with depth, and not the variation of Coriolis forces with latitude, that causes the wave motions. Theory suggests that the interaction of the turbulent eddies with these waves may channel the kinetic energy into horizontal shear flows, which would correspond to the observed zonal jets. Fig. 3 shows results from a numerical simulation using the anelastic approximation that illustrates the general possibility of this scenario[1].

By running a large number of simulations, we were able to investigate the dynamics in detail[1]. The jet thickness could be shown to follow a certain scaling law similar to classical Rhines' theory for beta-plane turbulence. Applied to the giant planets in the solar system, the observed number of jets is reasonable well predicted, which is remarkable given the simplicity of the model considered here.

7 Conclusions

Rotating convection is a fascinating fluid-dynamical phenomenon with large relevance in geo- and astrophysical systems, but it is still not well understood. Simulations on JUQUEEN have shown that the nature of the boundaries has a much larger impact than previously expected. In rapidly rotating systems, tiny Ekman boundary layers are shown to massively boost the heat transfer, contrary to common theoretical expectations. Also the processes of upscale kinetic energy transport and large-scale coherent structure formation appears to be strongly affected by the mechanical boundary conditions. Both effects need further study. We have also been able to demonstrate very clearly that intense alternating jets can be convectively driven by a complex interplay of compressibility effects and rotation. This should be an important effect in the interiors of gas planets, and must be considered as a possible explanation for the zonal winds observed on all giant planets in our solar system.

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